59 $v = c(r_0 - r)r^2$ $\frac{r_0}{2} \le r \le r_0$

maximize v

show $r = \frac{2}{3}r_0$

c = constant

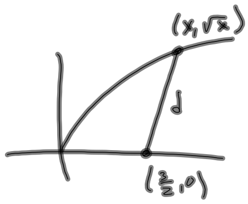
r_0 = initial radius (rest radius)

r = radius

v = velocity

Oct 21-8:52 AM

91



minimize d

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$d = \sqrt{(x-\frac{3}{2})^2 + (\sqrt{x}-0)^2}$

$d^2 = (\frac{3}{2}-x)^2 + (\sqrt{x}-0)^2$

endpt

$x=0$

$d = \frac{3}{2}$

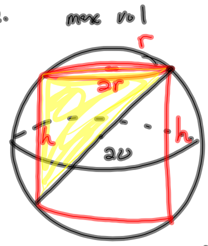
$d' = \frac{2(x-1)}{\sqrt{4x^2-8x+9}} = 0$

$x=1$

$\boxed{d = \frac{\sqrt{5}}{2}}$

Oct 21-9:35 AM

22. max vol



$V = \pi r^2 h$

$h^2 + (2r)^2 = 400$

$h^2 + 4r^2 = 400$

$r^2 = \frac{400-h^2}{4}$

$V = \pi\left(\frac{400-h^2}{4}\right)h$

$V' = -\pi\left(\frac{3x^2-400}{4}\right) = 0$ $x = \sqrt{\frac{400}{3}}$

$h = x = \frac{20\sqrt{3}}{3}$

$h=0$ $h=20$

$V=0$ $V=0$

Oct 21-9:43 AM

4.4c Modeling and Optimization

Examples from Economics

Maximum Profit: If there is a maximum profit, it occurs when marginal revenue = marginal cost

$P = R - C$

maximize Profit $P' = R' - C' = 0$

$R' = C'$

Oct 25-8:00 AM

Suppose r(x) = 9x and c(x) = x^3 - 6x^2 + 15x, where x represents 1000's of units. Is there a production level that maximizes profit? If so, what is it?

$r' = c'$

$9 = 3x^2 - 12x + 15$

$x = -(\sqrt{2} - 2) \approx .5858 \quad P = -1.656$

$x = \sqrt{2} + 2 \approx 3.4142 \quad P = 9.657$

$P = 9x - (x^3 - 6x^2 + 15x)$

$P' = 9 - (3x^2 - 12x + 15) = 0$

$x = .5858$

$x = 3.4142$

$x = 0$

$P = 0$

Oct 25-8:05 AM

average cost = c(x)/x $= \frac{c(x)}{x}$

Minimum Average Cost: If there is a minimum average cost, it occurs when average cost = marginal cost.

$A = \frac{c(x)}{x}$

minimize A $\quad A' = \frac{x \cdot c'(x) - c(x) \cdot 1}{x^2} = 0$

$x \cdot c'(x) - c(x) = 0$

$x \cdot c'(x) = c(x)$

$c'(x) = \frac{c(x)}{x}$

Oct 25-8:06 AM

Suppose c(x) = x^3 - 6x^2 +15x, where x represents 1000's of units. Is there a production level that minimizes average cost? If so, what is it?

$A = \frac{c(x)}{x} = \frac{x^3 - 6x^2 + 15x}{x} = x^2 - 6x + 15$

$\frac{x \cdot (3x^2 - 12x + 15) - (x^3 - 6x^2 + 15x)}{x^2} = 0$

~~$x = ? -1.1724$~~

$A' = 2x - 6 = 0$

$x = 3$

$A = 6_{min}$

$A'' = 2$

$+ +$

$x = 3 \quad A = 6$

$x = 0 \quad A = 15$

short cut

$x^2 - 6x + 15 = 3x^2 - 12x + 15$

$x = 0$

$A = 15$

$x = 3$

$A = 6$

Oct 25-8:08 AM

Oct 21-10:15 AM